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UNVEILING THE DIGITAL EQUATION THROUGH INNOVATIVE APPROACHES FOR TEACHING DISCRETE MATHEMATICS TO FUTURE COMPUTER SCIENCE EDUCATORS

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ABSTRACT

Aim/Purpose	This study seeks to present a learning model of discrete mathematics elements, elucidate the content of teaching, and validate the effectiveness of this learning in a digital education context.
Background	Teaching discrete mathematics in the realm of digital education poses challenges, particularly in crafting the optimal model, content, tools, and methods tailored for aspiring computer science teachers. The study draws from both a comprehensive review of relevant literature and the synthesis of the authors' pedagogical experiences.

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Methodology	The research utilized a system-activity approach and aligned with the State Educational Standard. It further integrated the theory of continuous education as its psychological and pedagogical foundation.
Contribution	A unique model for instructing discrete mathematics elements to future computer science educators has been proposed. This model is underpinned by informative, technological, and personal competencies, intertwined with the mathematical bedrock of computer science.
Findings	The study revealed the importance of holistic teaching of discrete mathematics elements for computer science teacher aspirants in line with the Informatics educational programs. An elective course, "Elements of Discrete Mathematics in Computer Science", comprising three modules, was outlined. Practical examples spotlighting elements of mathematical logic and graph theory of discrete mathematics in programming and computer science were showcased.
Recommendations for Practitioners	Future computer science educators should deeply integrate discrete mathematics elements in their teaching methodologies, especially when aligning with professional disciplines of the Informatics educational program.
Recommendations for Researchers	Further exploration is recommended on the seamless integration of discrete mathematics elements in diverse computer science curricula, optimizing for varied learning outcomes and student profiles.
Impact on Society	Enhancing the quality of teaching discrete mathematics to future computer science teachers can lead to better-educated professionals, driving advancements in the tech industry and contributing to societal progress.
Future Research	There is scope to explore the wider applications of the discrete mathematics elements model in varied computer science sub-disciplines, and its adaptability across different educational frameworks.

Keywords	digital education, discrete math, informatics, integration, professional competence

INTRODUCTION

The training of teaching staff in conditions of digital education implies the continuity of the constantly changing modern requirements of professional education and the new needs of general education. The task of organizing the subject training of the future teacher sets the following complex requirements: requirements of fundamentality, requirements of universality, integration, practical orientation, and necessity of harmonization in compliance with a specific professional standard (González et al., 2022). Regarding the problem of study, considering the problem of training future computer science teachers in the context of digital education, one of the conditions for solving modern problems of training future computer science teachers is the priority of choosing their subject-structural components based on the integrative-modular approach of the proposed educational programs. Success in such a situation will depend on the consistency and continuity of all links in the chain of professional training of future computer science teachers (Hartmann et al., 2023). An in-depth analysis of the possibilities of each studied subject area of the Informatics specialty, as well as an assessment of its internal fundamental and external integrative components, allows finding the common topics both for different subject areas at different levels of education, and for the structural components of one subject area. It also increases their effective mutual integration (Gravier & Ouvrier-Buffet, 2022). For example, the doctoral dissertation by Perminov (2019), can be noted. In addition, the study by Alfimova (2012) provides for a partial interdisciplinary study of discrete mathematics and computer science in terms of teaching the discrete mathematics elements using information and communication technologies for students of the natural-mathematical direction. Musinova (2001) studied the future computer science teachers in terms of teaching methods of discrete mathematics. The closest study area is the identification of various subject-professional competencies, which reflect the education quality within a certain content network distinguished in the general structure of the educational program (Kodzhaspirova, 2019; Qudaybergenova, 2021).

However, there remains a gap in the literature. The shift towards a new competence model of education necessitates an in-depth examination of the interlinkages among methodological, mathematical, psychological, and didactic facets of teaching discrete mathematics. Consequently, this research seeks to unveil a comprehensive model for instructing discrete mathematics elements. It aims to integrate them seamlessly with principal computer science subjects and validate the model's efficacy through experimental outcomes in the digital education sphere.

The study revealed that the problem of teaching the elements of discrete mathematics to future computer science teachers requires a special analysis of the conditions of the transition to a new competence model of education. This is conditioned by insufficient development of methodological, mathematical, psychological, and didactic aspects of the implementation of interdisciplinary connections, elements of professional disciplines, and discrete mathematics in the training process of future computer science teachers in the context of digital education. In this regard, the integrated teaching of the elements of discrete mathematics and the main subjects of computer science predefine the research relevance. Considering the above-mentioned facts, it is necessary to integrate the elements of discrete mathematics with the content of the main subjects of computer science. At the same time, there is a contradiction between the lack of methodology.

Thus, the research aims to present a model for teaching the elements of discrete mathematics, as well as the content of learning and to demonstrate the learning efficiency through the experiment results in the context of digital education.

LITERATURE REVIEW

Discrete mathematics holds a pivotal role as a foundation for computer science. Despite its significance, challenges persist in effectively imparting this subject to budding computer science educators. Recent research endeavors have shed light on pioneering pedagogical techniques, technological integration, and strategies for blending content to amplify the quality of discrete math instruction (Mazakov et al., 2021; Nurdaulet et al., 2018).

A notable body of work accentuates the integration of discrete mathematics with computer science while harnessing the power of modern educational technologies. Xia et al. (2023) carved a niche by devising adaptive teaching methodologies specifically tailored for undergraduate discrete math courses centered around computer science. Their insights reveal not just heightened student engagement but also superior conceptual clarity. Echoing this trend, Cao and Grabchak (2023) integrated the WebWorK online homework system in their discrete math syllabus, reporting substantial improvements in foundational skills and a shift towards personalized learning trajectories. Wang et al. (2023) brought in innovation by amalgamating blended learning with the interactive "rain classroom" framework, observing enhanced student enthusiasm and academic performance. Durcheva (2022) also provided a noteworthy contribution, advocating for integrating visualization tools, hands-on computing laboratories, and e-learning platforms into discrete math curricula.

Complementing this, other researchers have delved deep into weaving discrete mathematics across the fabric of the computer science curriculum, emphasizing its organic connection to programming. Lehmann (2022) stands out in this regard, championing the idea of rendering algorithms in discrete math more accessible through targeted programming drills. Tupouniua (2022) took an analytical approach, exploring students' revision patterns in algorithms when presented with counterexamples, unearthing intriguing findings on iterative refinement techniques. Drawing from seminal works, Jamaloodeen et al. (2020) emphasize innovative teaching strategies and applications that mirror real-world scenarios. Kodzhaspirova (2019) presents an overarching view of pedagogical methods, which becomes essential when broaching discrete mathematics.

From a technological perspective, Kuzmina et al. (2002) delve into the realm of creating educational websites, suggesting the potential for an integrated online platform for discrete mathematics. Meanwhile, Musinova (2001) shares methodologies tailored for future informatics teachers, emphasizing teaching discrete math concepts, backed by practical examples from St. Petersburg's academic context. Olenev et al. (2020) further enrich the discourse by discussing the untapped potential of the Maple computer algebra system in set theory and combinatorics education.

Furthermore, the work by Kuznetsov et al. (2016) offers a futuristic glimpse into the evolving roles and expectations of informatics educators, emphasizing the nexus between discrete math and modern teaching techniques. Romeo (2018) demonstrates the synthesis of programming with discrete math, providing hands-on applications and insights.

In summation, the contemporary literary landscape underscores a multi-dimensional approach to enhancing discrete math instruction for prospective computer science mentors. This encompasses content integration across computer science subjects, harnessing cutting-edge educational tools, forging ties with programming, and continually refining pedagogical expertise. In light of this rich tapestry of literature, our article seeks to build upon these foundational studies, offering fresh insights and empirical findings. Our contribution aims to further this discourse by providing innovative solutions and perspectives.

MATERIALS AND METHODS

The system-activity qualitative approach and the requirements of the State Educational Standard were considered in the research. An analysis of philosophical, psychological, pedagogical, and methodological literature on research problems was conducted. The analysis of textbooks and teaching manuals on mathematics and computer science, as the basic educational programs (EP) of school and university courses in computer science and mathematics, was also employed.

The fact that the theoretical foundations of computer science are taught in an integrated way with discrete mathematics was noted. A summary of the results of the authors' individual pedagogical experience obtained in the process of implementation of the EP in computer science was used as well. The theory of continuous education development formed the psychological and pedagogical basis of the study, and the directions of the professional and pedagogical concept of teaching the theoretical foundations of informatics to future teachers were adopted.

The theoretical foundation of the model for teaching the discrete mathematics elements in the conditions of digital education is offered based on educational program analysis and the division of subject-professional competencies into the informative (knowledge of the elements of special discrete mathematics), technological (mastery of the professional activity methods), and personal (mastery of professionally significant personal qualities). The participants were 157 students in the quasi-experiment, with 77 in the experimental group undertaking the integrated course, and 78 in the control group undergoing traditional learning. Textbooks and teaching manuals were reviewed, alongside computer science and math curricula and the authors' pedagogical experiences. Resources included material objects for the organization and implementation of the pedagogical phenomenon. Digital resources and software (Python, Maple, MathCAD, electronic educational and methodological complex) were used (Olenev et al., 2020; Romeo, 2018). They were compared and combined to achieve goals as the means of forming the discrete mathematics elements for students, a reliable methodological organization of the process that can complement each other. Continuous education theory formed the psychological-pedagogical basis. A theoretical model for teaching discrete math elements based on competencies was proposed. An elective course integrating elements across computer architecture, programming, and networks was designed. Knowledge assessment was conducted before and after experimental implementation.

The research in the context of digital education aims to study fundamental topics, which make up the bulk of the content of teaching in subjects that will be transferred to the future computer science teacher. This is the most important task of the learning process. In this regard, it is necessary to analyze its conceptual part to conclude on the content of a subject. Thus, in the process of training future computer science teachers, one of the main problems is the focus on mathematical knowledge in the conceptual part of the basic disciplines of this specialty, including the discrete mathematics elements. This, in turn, requires the creation of a methodology of integrated teaching of the elements of discrete mathematics (in the process of teaching the future computer science pedagogues) concerning the subjects that make up the theoretical foundations of computer science.

RESULTS

The following aims in the training of a computer science teacher within the content line on the elements of discrete mathematics (EDM) were defined: knowledge of the original sections of discrete mathematics; basic ideas and methods of discrete analysis; the use of the discrete mathematical models to solve theoretical and practical problems; analysis and interpretation of the results obtained; knowledge of the history of the discrete mathematics development; comprehension of the place of discrete mathematics in the general structure of scientific knowledge; knowledge of the interrelation between informatics and discrete mathematics, the role of discrete analysis in the development of modern technologies; and mastering the mathematical content of the discrete component of school courses in computer science (content). Also considered were: the knowledge of the basic methods of discrete analysis; implementation of classical computational and combinatorial algorithms; solution of the simplest discrete mathematics problems; preparation for the search and application of rational methods of discrete analysis in solving professional and practical problems; the ability to compose the mathematical content of the discrete component of the school course of informatics; and possession of the basic methods for solving discrete mathematical problems of the school course of informatics (technological). Therefore, the training of future computer science teachers should consider the content of EDM (not specified in the educational program), especially the following requirements:

1. The basics, namely analysis of scientific literature, and the three-block structure of students on the elements of discrete mathematics (informative, technological, personal).
2. Following the content formed on the EDM basis, given in the educational program in the specialty “Informatics”, subject-professional competencies have important foundations.
3. Professional competence is the main goal in training the future computer science teacher. On this basis, the authors developed the training program for the elective course “Elements of Discrete Mathematics in Computer Science.” The elective course “Elements of Discrete Mathematics in Computer Science” includes three modules: logical foundations of computer architecture; mathematical foundations of algorithmization and programming; and elements of graph theory in computer networks.

To develop a model for teaching discrete mathematics to a future computer science teacher in the conditions of digital education, it is necessary to define the concept of Kodzhaspirova (2019). A model is a created artificial object in the form of a drawing/table, that represents in the simplest form and reproduces the correlations, properties, and structure between the elements of the investigated object, similar to the studied object. The model has only the necessary degree that resembles any real object; that is, it expresses the researcher’s point of view within the framework of the study object. Thus, regarding the development of the model for teaching elements of discrete mathematics, the authors indicate possible options for solving the problem.

A model is a real physical object or process, theoretical structure, or an information image that reflects certain properties of the studied object, process, or phenomenon. Analyzing the goals and ways of implementing the EDM teaching model for a future computer science teacher in the conditions of digital education, focusing on the theoretical principles of building a model, the fundamental, successive, integration, career guidance, and modular principles when developing a system of goals in the study was used as a basis. Since the process of forming the qualifications of a future computer science teacher in the conditions of digital education aims to develop the universal and professional competencies of a specialist, the authors focus on the main features of the competency-based approach in discrete mathematics. The model is proposed to a future computer science teacher in the conditions of digital education following the design of the "main" goals of teaching discrete mathematics (Figure 1).

At the main stage of general EDM education, subject-professional competencies are specified by the formulation of "specific" goals – the structure of professional competence of the module sections that make up the content of general education in discrete mathematics. As for the subject-professional competencies of in-depth EDM training, they are specified by one or another variable component (disciplines for choice of themes on EDM, teaching and research activities of students, etc.). The content of the training program of the elective course "Elements of Discrete Mathematics in Computer Science" for future computer science teachers is presented in Table 1.

The digital resource "Discrete Math" (DM) stands out as a potential educational tool for teaching programming languages, such as Python. This resource allows solving problems on this topic online (Web 4.0) in the Python programming language, providing the theoretical foundations of the discrete mathematics elements. DM content is divided into the following parts: theoretical, practical, and knowledge control system. The DM goal is to study the elements of discrete mathematics in combination with the Python programming language, i.e., an electronic textbook intended for the teacher for the complete assimilation of the material in the educational process. DM helps to make educational material a visual one, improves students' understanding of programming tasks, as well as involves, not only mastering the theoretical knowledge that is necessary for professional activities, but also its practical application.

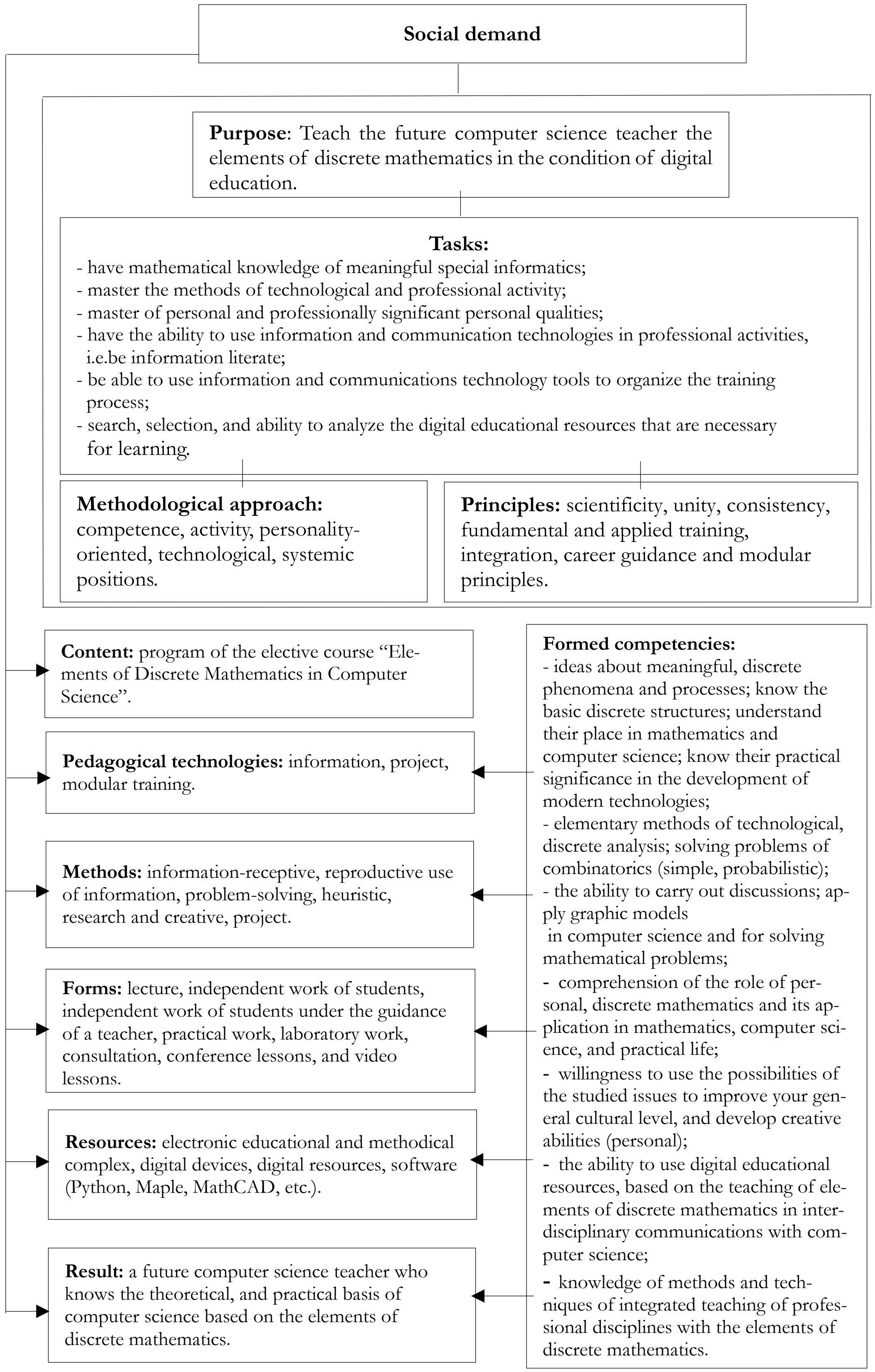


Figure 1. Model of teaching EDM to a future computer science teacher in the condition of digital education

Table 1. The content of the training program "Elements of Discrete Mathematics in Computer Science" for future computer science teachers

No.	Title of the Topics	Content
Module I. Logical foundations of computer architecture		
1	Statements and logical operations	Formulas of the algebraic logic. Computer logical elements. Functions of the algebraic logic. Methods for setting the logical functions. Equivalence of formulas. Boolean algebra.
2-3	Disjunctive and conjunctive normal forms (DNF and CNF)	Disjunctive and conjunctive normal forms. Minimization of Boolean functions in the disjunctive normal form class. Karnaugh maps.
4-5	Boolean algebra and set theory. Switching schemes	Boolean algebra and set theory. Switching schemes. Drawing up the computer electronic circuits.
Module II. Mathematical foundations of algorithmization and programming		
6-7	Elements of set theory	Ways to transfer sets. Operations on sets. The direct product of sets. Partitions and coverings of sets. Basic properties of matrices of binary relations. Relationship properties. Equivalence relation. Order relation. Lexicographic order. Functional relationships (functions).
8-9	Elements of combinatorics	Sum and product rules. Placements, combinations, partitions, and permutations with repetitions. Examples of applying elements of combinatorics to sets and information protection.
Module III. Elements of graph theory in computer networks		
10-11	Basic concepts and definitions	Methods for setting graphs. Graph isomorphism. Subgraphs. Operations on graphs.
12	Paths, reachability, connectivity	Definition of connected components and strongly connected components. Exploring graph paths.
13	Eulerian and Hamiltonian graphs	Eulerian graphs. Construction of Eulerian cycles. Hamiltonian graphs.
14	Distances in graphs, weighted graphs	Distance in a graph. Finding the shortest paths. Flows in networks.
15	Trees, forest. Minimum spanning trees	Trees and forest. Defining a minimum weight spanning tree.

In general, the digital educational resources in question are no different from e-learning tools. Therefore, electronic learning tools (ELT) or DER is an electronic tool with systematized material that provides a creative and active assimilation of knowledge by pupils and students following science and practice in the field of education. ELT should be at a high level, both in artistic and performance terms regarding information completeness, quality of teaching aids, quality of technological performance, clarity, logic, and coherence of thought (Horváthová et al., 2022). Considering the basic requirements for e-learning tools, the ELT structure is defined following the identified content. This ELT can also be used for distance learning. Elements of discrete mathematics in computer science: practice, tasks, theory.

1. *Theory:* Module 1 – mathematical foundations of computer architecture; Module 2 – mathematical foundations of algorithmization and programming; Module 3 – graph theory elements in computer networks.
2. *Practice:* types of digital resources (Microsoft programs, electronic textbook, cloud services) used in the theoretical interpretation of the discrete mathematics elements; types of digital resources (platforms, cloud services) where distance learning with the discrete mathematics elements can be conducted; types of digital resources (Microsoft programs, electronic textbook,

cloud services) where the distance learning with the discrete mathematics elements can be conducted.

3. *Monitoring:* students' independent work; test tasks; reflection.
4. *References:* design work; glossary; literature, digital resources.

Forming the identified goals for general education on the discrete mathematics elements, it can be said that students should have the following level of knowledge on the discrete mathematics elements: formation of ideas about discrete phenomena and processes, knowledge of the basic discrete structures, understanding of their place in computer science, practical significance in the development of modern technologies (content); skills of the simplest methods of discrete analysis, solve problems of combinatorics elements, carry out the simplest probabilistic judgments, apply graphical models in solving problems (technological) in computer science; understand the role of discrete mathematics in computer science and its application in practical life; willingness to use the possibilities of studied problems to improve their general cultural level, the development of creative abilities (personal). Analysis of the literature relating to EDM teaching revealed numerous views and opinions on the teaching methods. Each method provides a special type of educational activity for the teacher and the cognitive activity of students. At the same time, it helps master a certain type of education.

Forms of organizing training in the formation of EDM knowledge in the training of future computer science teachers is an externally coordinated activity of the teacher and students, carried out in a set sequence. They are classified based on the following criteria. It is recommended to organize them by the number of students (collective, group, individual), by place of study, etc. Analysis of the sources of scientific literature on this issue showed that the terms "forms of organization of learning" are debatable and establish only one form, for example, types of training sessions – lesson, lecture, student's work with a teacher, student's independent work, and tour. Several scientists considered them as the forms of organization of current training, others considered them as organizational forms of education. The following forms were used in the experiment: lecture, practical work, laboratory work, independent work of a student, independent work of a student with a teacher, consultations, conference lessons, and video lessons.

To teach EDM using the digital educational resource discussed above, let us consider the practical application of mathematical logic in programming. When defining logic operations, it is possible to determine the truth of any formula using the following table, i.e., it is possible to build a truth table for them (Table 2).

Table 2. The truth table of logical operations

φ	ω	$\varphi \wedge \omega$	$\varphi \vee \omega$	$\boldsymbol{\varphi \to \omega}$	$\varphi \leftrightarrow \omega$	$\varphi \mid \omega$	$\varphi \downarrow \omega$	$\varphi \oplus \omega$
0	0	0	0	1	1	1	1	0
0	1	0	1	1	0	1	0	1
1	0	0	1	0	0	1	0	1
1	1	1	1	1	1	0	0	0

An elementary conjunction (disjunction) is a conjunction (disjunction) of variables or their negations, and a disjunctive normal form (DNF) is a disjunction of elementary conjunctions. Conjunctive normal form (CNF) is a conjunction of elementary disjunctions. Example 1:

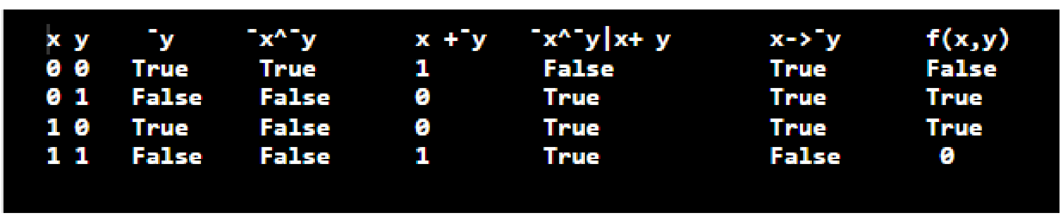
$$f(x, y) = \big((\bar{x} \wedge \bar{y})\big|(x \oplus \bar{y})\big)(x \to \bar{y}). \tag{1}$$

Let us consider the use of elements of mathematical logic in the example of a function. In mathematical logic, an algebraic sentence is a sentence formulated using equations between terms with free variables. This function includes 9 actions. Using a given function, it is necessary to bring the truth table, the function into conjunctive and disjunctive normal form as a direct application of mathematical logic. For this function, it is necessary to construct a truth table: $f(x,y) = \left((\bar{x}\wedge\bar{y})\middle|(x\oplus\bar{y})\right)(x \to \bar{y})$. Solution: construct a truth table with all possible sets of values of the arguments $x_1, x_2, \ldots, x_n$, on its left side, and on the right side, a column of f values that correspond to these sets. The number of all sets 0 and 1 of a function of n variables will be equal to 2^n, i.e., for a function of one variable – $2^1 = 2$, for two – $2^2 = 4$, for three – $2^3 = 8$, etc. All operations on variables x and y (conjunction, disjunction, direct sum, implication) can be replaced with ready-made values of a Boolean function. The function is represented through a truth table. An example of converting a function to the numbers 0 and 1 is presented in Table 3.

Table 3. The truth table for the function $f(x,y) = \left((\bar{x}\wedge\bar{y})\middle|(x\oplus\bar{y})\right)(x \to \bar{y})$

x	y	$\bar{y}$	$\bar{x}\wedge\bar{y}$	$x\oplus\bar{y}$	$(\bar{x}\wedge\bar{y})\|(x\oplus\bar{y})$	$x \to \bar{y}$	$f(x,y)$
0	0	1	1	1	0	1	0
0	1	0	0	0	1	1	1
1	0	1	0	0	1	1	1
1	1	0	0	1	1	0	0

Many programming languages provide the ability to use variables that store Boolean values (true or false). In the Python programming language, such variables can be True or False. The disjunction statement is the OR operator in Python, while the conjunction statement is the AND operator in Python. The result of programming elements of mathematical logic in the Python programming language (Figure 2).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x y    ¯y      ¯x^¯y      x +¯y   ¯x^¯y|x+ y      x->¯y      f(x,y)
0 0  True     True        1       False           True       False
0 1  False    False       0       True            True       True
1 0  True     False       0       True            True       True
1 1  False    False       1       True            False       0
```

Figure 2. The result of program execution in Python

Example 2. Reduce the given function: $f(x,y) = \left((\bar{x}\wedge\bar{y})\middle|(x\oplus\bar{y})\right)(x \to \bar{y})$ to disjunctive and conjunctive normal forms (DNF and CNF). Solution: let us reduce the formula: $f(x,y) = \left((\bar{x}\wedge\bar{y})\middle|(x\oplus\bar{y})\right)(x \to \bar{y})$ to DNF.

$$x|y = \overline{x\wedge y},\ x\oplus \mathrm{y} = \overline{x \leftrightarrow y},\ x \leftrightarrow y = xyv\bar{x}\bar{y} \tag{2}$$

formulas:

$$\overline{x\vee y} = \bar{x}\wedge\bar{y} \tag{3}$$

apply Morgan's law:

$$x \to y = \bar{x}\vee y, x\wedge\bar{x} = 0, x\vee\bar{x} = 1, \tag{4}$$

using the formulas of the third law, the authors reduced the formula to DNF:

$$f(x,y) = \left((\bar{x}\wedge\bar{y})\middle|(x\oplus\bar{y})\right)(x \to \bar{y}) = \ \bar{x}y\vee x\bar{y} \tag{5}$$

With the help of elementary transformation, using the properties of a logical operation, it was reduced to a formula to a conjunctive normal form: $f(x, y) = \big((\bar{x} \wedge \bar{y}) \big| (x \oplus \bar{y})\big)(x \to \bar{y}) = \bar{x}y \vee x\bar{y}$ – CNF.

Elements of Graph Theory in Computer Networks

The graph properties are also considered in determining the content when creating digital educational resources, Internet resources, and site maps. The Internet can be represented as a graph, where the graph is Internet sites, and the walls are links (hyperlinks) from one site to another. The web graph (Internet) is a link that consists of billions of ceilings and walls that are constantly being added and changed. However, the Internet has a mathematical structure with constant properties that obey graph theory. In some problems of applied mathematics, communication systems between several objects are studied. Objects are called ceilings, indicated by a dot, and the connection between these ceilings is called walls and is indicated by placing arrows between the corresponding dots. Such systems form a graph, for example, the system of streets in a city (Kuzhel et al., 2013). The graph's roofs are the intersections of the streets, and the walls are the streets, the direction of which is known; electrical system; molecules in a chemical bond; geographic map; the connection between people (pedigree); block diagram of the program; transport, airlines (Bekenov et al., 2022). If there are several paths from one point to another, then it is needed to look for the shortest and the most convenient one. To solve such problems, it is necessary to calculate graphs.

Suppose that graph G(V, E) is given. A label (or label distribution) of a graph is a pair of functions: f: V→ and g: E→ S_E, where S_v, S_E – the label sets of vertices and edges (arcs). The quadruple (V, E, f, g) is called a weighted or labeled graph. If v∈V, then f(v) is the weight of the vertex v, if e∈E, then g(e) is the weight of the arc e. Often, only vertices or only edges (arcs) are marked. Figure 3 shows a labeled graph (V, E, f, g) representing a map of highways with their length (Aloev et al., 2021).

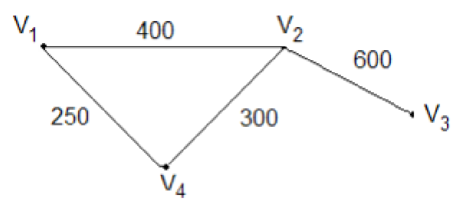

Figure 3. Weighted graph

$$V = \{V_1, V_2, V_3, V_4, V_i\},\ E = \{\{V_1, V_2\}, \{V_2, V_3\}, \{V_1, V_4\}, \{V_2, V_4\}\} \tag{6}$$

where f: V → S_v: S_v – set of cities, f(V_1) = Astana, f(V_2) = Ekibastuz, f(V_3) = Semipalatinsk, f(V_4) = Karaganda. g: E → S_E: S_E – set of distances, g({V_1, V_2}) = 400, g({V_2, V_3}) = 600, g({V_1, V_4}) = 250, g({V_2, V_4}) = 300. Information about the weights of arcs in a weighted graph can be represented as a weight matrix W = (w_{ij}) where w_{ij} – the weight of the arc (v_i,v_j), if this arc exists. If it does not exist, then the corresponding weight is denoted by 0 or ∞ depending on the applications (∞ - if the vertices are not adjacent). For a graph from a specific example, the matrix is as follows (Figure 4).

$$W = \begin{pmatrix} 0 & 400 & \infty & 250 \\ 400 & 0 & 600 & 300 \\ \infty & 600 & 0 & \infty \\ 250 & 300 & \infty & 0 \end{pmatrix}$$

Figure 4. Matrix for the graph:
V = {V1, V2, V3, V4, V$_i$}, E = {{V1, V2}, {V2, V3}, {V1, V4}, {V2, V4}}

The concepts of distance, radius, eccentricity, and others for weighted graphs are also introduced. Suppose that G(V, E) is a weighted graph in which the weight of each arc (v_i,v_j) is a certain number μ (v_1,v_2), μ (v_1,v_2) ∈ V. Then, the weight of the path (v_1↔ v_{n+1}) is the number:

$$\mu = \sum_{i=1}^{n} \mu(v_i, v_{i+1}); \quad (7)$$

the weighted distance $d_w(v_1, v_{n+1})$ between vertices v_1 and v_{n+1} is the minimum of the weights v_1↔ v_{n+1} of paths; the shortest path between v_1 and v_{n+1} is a path whose weight is equal to $d_w(v_1, v_{n+1})$; the weighted eccentricity of the vertex v (denoted as (e_w(v)) is the number:

$$ew(v) = max\{dw(v,u) / \ U \in V\}. \quad (8)$$

The weighted central vertex of the graph G is the vertex v for which:

$$ew(v) = min\{\mathrm{e}w(u)/u \in V\}, \quad (9)$$

and the weighted radius of the graph G (denoted r_w(G)) is the weighted eccentricity of the central vertex.

FINDING THE SHORTEST PATHS

Moreover, there are various ways (algorithms) to determine the shortest paths. The choice of algorithm depends on the conditions of the problem, the method of solution (manually or via computer), and other factors. For weighted graphs with non-negative weights of all edges, Dijkstra's algorithm (Anderson, 2004) is used. This algorithm can be found in various versions, for example, with and without the use of a weight matrix. Suppose that G(V, E) – a weighted graph with n vertices and W = (w_{ij}), – its weight matrix w_{ij}>0. For example, it is required to find the weighted distance from a fixed vertex v_i (source) to some other vertex. Let's denote T_1 the set of vertices V/{v_1}, т.е. T_1 – is the set of all vertices of V without a source. Let's denote $D^{(1)} = (d_1^{(1)}, d_2^{(1)}, \ldots, d_n^{(1)})$, where $d_i^{(1)} = 0$, $d_j^{(1)} = w_{ij}$ (i≠j), i.e. $D^{(1)}$ – is the i row in the weight matrix. Suppose at step S the sets of vertices T_s and string $D^{(s)} = (d_1^{(s)}, d_2^{(s)}, \ldots, d_n^{(s)})$ are defined. Now, the task is to pass from T_s to T_{s+1}- and from $D^{(s)}$ to $D^{(s+1)}$. Choose a vertex v_j in T_s such as:

$$dj(s) = min\{dk(s) \ / \ vk \in Ts\}. \quad (10)$$

Denote:

$$T_{s+1} = T_s \backslash \{v_j\}, D^{(s+1)} = \left(d_1^{(s+1)}, \ldots, d_n^{(s+1)}\right) \quad (11)$$

where:

$$\begin{cases} \mathrm{d}_\mathrm{k}^{(s+1)} = \mathrm{d}_\mathrm{k}^{(s)}, \text{if } \mathrm{v_k} \notin \mathrm{T_{s+1}}, \\ \mathrm{d}_\mathrm{k}^{(s+1)} = \min\left\{\mathrm{d}_\mathrm{k}^{(s)}, \mathrm{d}_\mathrm{j}^{(s)} + \mathrm{w_{jk}}\right\}, \text{if } \mathrm{v_k} \in \mathrm{T_{s+1}} \end{cases} \quad (12)$$

At step S = n-1, the string $D^{(n-1)} = (d_1^{(n-1)}, d_2^{(n-1)}, \ldots, d_n^{(n-1)})$ is true, in which $d_j^{(n-1)}$ is equal to weighted distance from vertex v_i to vertex v_j: $d_j^{(n-1)} = d_w(v_i, v_j)$. Consider the practical application of Dijkstra's algorithm for the problem of finding the shortest path in a network. Suppose it is required to find the shortest weighted distances from vertex 1 (source) to the rest of the vertices (Figure 5).

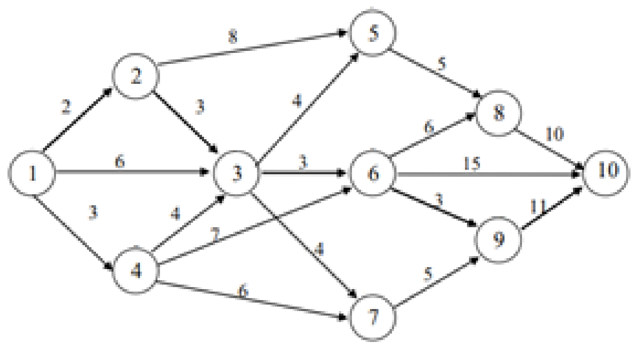

Figure 5. Weighted graph

Basic concepts of networks: nodes (in the graph: vertices), communications (in the graph: edges). Any network consists of individual participants (people or objects in the network) and relationships between them. Lines are often displayed through graphical structures consisting of multiple vertices and edges. Participants are represented as network nodes, and their relationships are represented as lines connecting them. Such visualization helps to evaluate networks qualitatively and quantitatively. For this graph, the distance is determined through the weight matrix: V={1,2,3,4,5,6,7,8,9,10}– the set of vertices. For this graph $d_w(1.1) = 0$, $d_w(1.2) = 2$, $d_w(1.3) = 5$, $d_w(1.4) = 3$, $d_w(1.5) = 9$, $d_w(1.6) = 8$, $d_w(1.7) = 9$, $d_w(1.8) = 14$, $d_w(1.9) = 11$, $d_w(1.10) = 22$, the distance is determined through the weight matrix (Figure 6).

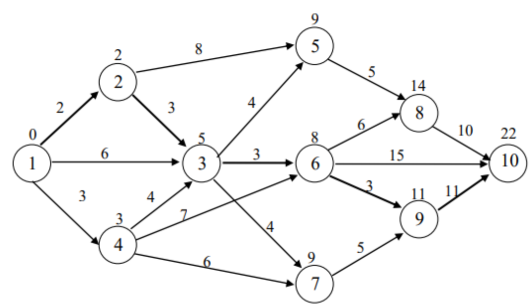

Figure 6. Weighted graph

Now, let's solve the problem of finding the shortest path in the network using the algorithm of dynamic programming. In this task, the algorithm consists of 4 steps: the results of the algorithm from 1 to 2 – 2, to 3 – 5, to 6 – 8, to 9 – 11, to 10 – 22, the shortest path is approximately as follows: 1→2→3→6→9→10.

According to the results of the above-stated research, the teaching methodology was implemented in the process of training future computer science teachers. Now, let us focus on the results of the pedagogical experiment conducted on this training. For this purpose, an experiment was carried out at the Kazakh National Women's Teacher Training University. The participants in the experiment were 157 students, 77 students visited the experimental group, and 78 students visited the control group. An elective course was introduced into the experimental group according to the proposed model for teaching the discrete mathematics elements to future computer science teachers. For the control group, it was offered training according to the previous educational program. Students in the experimental group were given priority to several skills used in the main part. The level of student progress in the control group was monitored during the classes. The results of observations are presented in Figure 8.

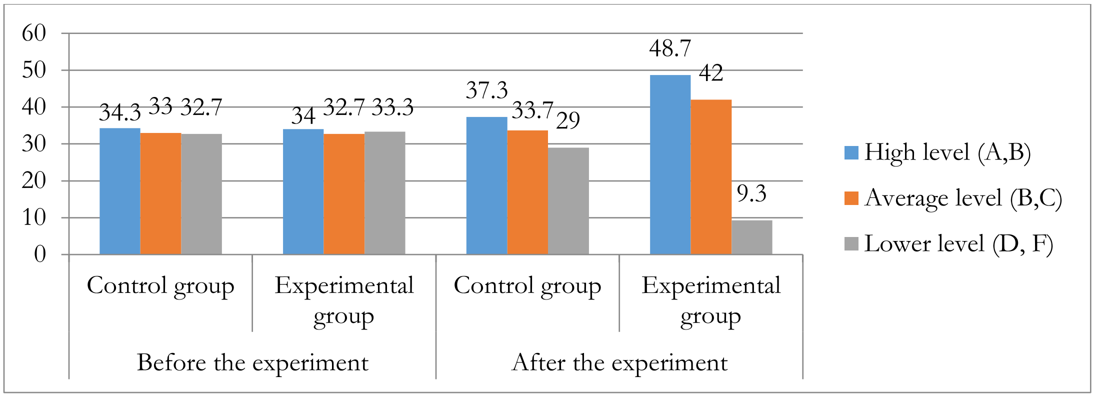

Figure 8. Comparative indicator of the knowledge level of a future computer science teacher by elements of discrete mathematics

Figure 8 shows that the effectiveness of using the model of teaching elements of discrete mathematics by a future computer science teacher indicates an increase in the knowledge level of students in the experimental group. In the experimental group, the high level of assessment (A, B) increased from 34% to 48.7%, the average level (B, C) increased from 32.7% to 42%, the low level (D, F) decreased from 33.3% to 9.3%. As for the students of the control group, their knowledge level has at least slightly increased even without the use of the learning model, which is a good indicator since continuing the traditional training program. When comparing the indicators of high and medium levels in the experimental group and the control one, then (9.8) prevails in the experimental group. This proves the advantages of the proposed learning model. It can be concluded that the application of the learning model gives positive results. The experiment results showed that the high level of knowledge of the future computer science teacher on the elements of discrete mathematics demonstrated the effectiveness of the proposed model. These quantitative and qualitative findings confirm the pedagogical experiment's effectiveness in validating the proposed model of teaching discrete math elements to aspiring computer science teachers.

DISCUSSION

Through the concepts of information technology for the formation of competencies reflected in the learning model, it is possible to form digital educational resources and develop the teaching of discrete mathematics elements in training of the future computer science teachers following modern requirements. A digital educational resource (DER) is a set of digital data created or designed for a specific education that is used in the educational process (Kuzmina et al., 2002). The use of digital educational resources in the field of education qualitatively changes the content, methods, and organizational forms of teaching for teachers. The tasks presented in a digital form allow students to use time efficiently, determine the necessary resources, search, and compare the solutions found, and choose, and justify a rational way for solving the problem. The use of digital educational resources in the process of general learning can take various forms: interactive, multimedia resources, modeling of specific resources and processes for research purposes, communicative – the ability to communicate directly, productive – operations automation (Bakoev, 2021). The use of digital educational resources in teaching the elements of discrete mathematics expands the interdisciplinary connections of computer science and mathematics, and the integration of subject knowledge when performing tasks contributes to the creative activity of students. The use of digital educational resources in teaching increases the creative activity and quality of students' knowledge (Msosa et al., 2022).

In the network processing of a digital educational resource in computer science, the structure of the implemented material must be considered. A hierarchical information structure can be considered in the creation of means of education informatization. Grinshkun (2004) defined the term "electronic hierarchy" as a hierarchical structure of any data that is created, presented, and distributed using information and communication technologies. The ability to build a hierarchy in the field of education contributes to teaching students using a creative approach (the ability to construct using various criteria, hierarchically organize data, objects, and facts known to them, as well as choose the hierarchy levels) (Bostanov, 2010; Ead et al., 2021).

The results of this study validate the effectiveness of the proposed integrated discrete mathematics learning model in boosting conceptual understanding and applied skills among computer science teacher aspirants. The knowledge gains are consistent with prior research advocating strategic blending of discrete math instruction with core computer science subjects and technologies.

Building on the works of Xia et al. (2023) and Wang et al. (2023), the research emphasizes adaptive learning strategies specific to computer science. Both these previous studies spotlighted the merits of blending traditional educational approaches with adaptive models, especially when navigating the complexities of computer science. Our study takes their foundational ideas a step further, illustrating the tangible benefits this blended approach can confer upon teacher aspirants.

The tech-centric components of our model draw inspiration from Durcheva (2022) and Duan et al. (2022), who have both advocated for the intensive integration of computational tools. Their assertions about the importance of hands-on programming experiences in reinforcing discrete mathematics concepts align seamlessly with our findings. The tangible, real-world application of theoretical concepts underscores the importance of this integration.

Greefrath et al. (2022) have highlighted the value of modeling-based strategies, emphasizing application-oriented learning. Similarly, Fulton et al. (2022) promote bridging connections with traditional school math practices. Our research concretely manifests these ideas by showcasing how leveraging real-world applications and examples can deepen students' understanding and retention of core concepts.

From a content perspective, the emphasis on mathematical logic, graph theory, and combinatorics connects with Wu et al.'s (2023) and Ferrarello et al.'s (2022) recommendations to apply these topics for structural understanding.

Echoing the sentiments of Burroughs et al. (2023) and Matitaputty et al. (2022), our research underscores the importance of cultivating discrete math pedagogical knowledge. The structured approach to discrete mathematics, combined with pedagogical strategies, presents a holistic framework for future educators. Currently, the study of teaching EDM is an important part of mathematics education at all levels of education and for all students (Kuznetsov et al., 2016; Perminov, 2012).

In the proposed model of teaching elements of discrete mathematics during the training process of the future computer science teacher, the main attention was paid to the general teaching of discrete mathematics elements within the framework of a bachelor's degree in pedagogical education. Thus, in training computer science educators, the EDM (Electronic Discrete Mathematics) curriculum is seamlessly integrated into various professional disciplines (Bostanov & Suranchiyeva, 2021). Trainees learn to grasp and apply computer hardware and system software knowledge. They develop algorithms and create computer programs and mobile applications to tackle practical issues, including those related to robotics programming. Their education includes applied mathematics, computer and mathematical modelling, which are essential in their professional toolkit. Furthermore, students acquire knowledge about information theory, combinatorics, mathematical logic, and algorithmic graphical representation techniques. Python is utilized specifically for developing graphical computer games with libraries like Pygame (Sandefur et al., 2022). The training also encompasses programming for mobile devices and services, along with understanding the methods and tools for information and intellectual property protection. Concepts such as data "secrecy" and "integrity", as well as security practices like data backup and encryption, are also covered. The use of Digital Educational Resources (DER) is encouraged to foster an interdisciplinary approach, linking EDM with informatics.

This approach also includes methods and techniques for the integrated teaching of professional informatics disciplines within the EDM framework (Babak et al., 2021; Shynkariuk, 2022).

CONCLUSIONS

The pedagogical experiment validated the effectiveness of the proposed discrete mathematics teaching model for computer science teacher aspirants, as evidenced by the knowledge growth in Table 1 and Figure 1. Students gained a robust understanding of logical operations, normal forms, switching circuits, set theory, combinatorics, and graph concepts like connectivity and spanning trees. They exhibited enhanced skills in applying mathematical logic, solving basic discrete problems, representing algorithms graphically, and determining shortest paths in networks. The model develops the key competencies defined through educational program analysis – conceptual knowledge of discrete math elements, technological expertise in computer science, and related personal qualities. Overall, the model emphasizes competencies across informative, technological, and personal domains. The informative aspect entails discrete math knowledge, while the technological aspect covers computer science skills and methods. The personal aspect involves professional qualities significant for computer science educators. This multidimensional framework helps train well-rounded teachers.

The study revealed that the problem of teaching the elements of discrete mathematics to future computer science teachers requires a special analysis of the conditions of the transition to a new competence model of education. All this is due to insufficient development of methodological, mathematical, psychological, and didactic aspects of the implementation of interdisciplinary connections, elements of professional disciplines, and discrete mathematics in the training of future computer science teachers in the context of digital education. Considering the above-mentioned facts, it is necessary to integrate the elements of discrete mathematics with the content of the main subjects of computer science. At the same time, there is a contradiction between the lack of a methodology for teaching it.

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